Available online on www.shts.org.rs, www.cee.rs

Comput. Ecol. Eng. 4(1)(2008)8-15 ISSN 1452-0729
Scientific paper

A Multivariable and multilevel fuzzy logic control system

Jelenka B.Savković-Stevanović
University of Belgrade, Faculty of Technology and Metallurgy,
11000 Belgrade, Karnegijeva 4, Serbia
e-mail:savkovic@tmf.bg.ac.rs

(Receieved 13 July 2008, accepted 23 July,2008)

Abstract

In this paper higher order multilevel fuzzy control system was illustrated. A tuning method with second and third order membership functions was developed. The higher order fuzzy multilevel control was made to vary with respect to the plant's normalized dead time and system state error. As a case study it is used the plant of two coupled tanks. Simulation and experiments are presented to show validity of the proposed tuning methods. The obtained results have shown the advantages of the higher order multilevel fuzzy logic control in robustness and parameter selectivity. The higher order multilevel controller was settled faster by the first order multilevel fuzzy logic controller in conjunctions with less overshoot in set point control. The obtained results were illustrated feasibility of using the third order multilevel fuzzy controller to represent tuning formula.

Keywords: Multilevel fuzzy control, higher order fuzzy controller, normalized dead time, state error.

1. Introduction

The multilevel fuzzy functions are very useful for phase plane based analysis which was carried out, showed that the multilevel modified tuning actually made the fuzzy logic control able to adapt the control environment (Xu et.al.,1996; (Savković-Stevanović, 2004; Savković-Stevanović et.al.,2000(a); Hang et.a.,1994, Ho et.al.,1995; Chunsheng et.al.,1988; Savković-Stevanović et. al. 2000(b) and Savković-Stevanović, 2000). Also, the multilevel fuzzy functions were applied to different kinds of faults classification and recognition (Savković-Stevanović, 2000).

In the design method based on gain and phase margins, it is also important to select a suitable equivalent gain/phase margin contour so as to obtain appreciable performance. In previous studies it was found that is difficult to select such as a contour, and the improper allocation of the equivalent contour will degrade the system's performance (Savković-Stevanović et.al.,2000(a); Chunsheng et.al.,1988).

Thus, some guidelines for designing a fuzzy logic controller have been developed and theoretically proven. These rules and formula are helpful in eliminating the most time consuming trial-and-error procedures in the synthesis and design of fuzzy control systems.

A control effort will be partly or fully saturated outside the universe of discourse. In general, fuzzy logic control systems may have better system performance but the complexity of the fuzzy rules base and the additional degree of freedom increase the difficulty of design.

This paper demonstrates higher order multilevel fuzzy control system. The higher order multilevel fuzzy logic controller-HMFLC shows the best performance in comparing with multilevel fuzzy logic controller-MFLC, simple fuzzy logic controller-FLC and conventional proportional-integral -PI controller with less overshoot in set point control. As a case study the plant of two coupled tanks was used. The improvement in system performance is confirmed through both simulation and experimental results. The obtained results have shown the advantages of the higher order multilevel fuzzy logic controller in robustness and parameter selectivity for nonlinear systems. The higher order multilevel fuzzy controller was used to achieve an improvement in the performance of the auto controller tuning. A tuning method with higher order multilevel membership function is incorporated in a tuning formula based on the gains/phase margins.

2. Fuzzy logic

Unlike binary logic, fuzzy logic do not restrict a variable to be a member of a single set, but recognize that a given value may fit to varying degrees into several. Fuzzy systems operate by linguistic variables classification such as big, high, slow, medium, near zero, or fast, and testing variables with if-then rules, which produce appropriate responses. Each rules then weighted by degree of fulfillment of the rule invoked, this is a number between 0 and 1, and may be thought of as probability that a given number is considered to be included in a particular set. A wide variety of shapes is possible fulfillment functions, with triangles and trapezoids being the most popular. Fulfillment functions can be in the form:

$$\mu(x,\mu,s,p) = (\exp(-(|x-\mu|)^{p}/s) \qquad (1)$$

here μ, s, and p are user chosen parameters and x is the values to be tested. The function was chosen because of its flexibility, by changing μ, s, and p whole families of different function can be obtained. For p=2 this is an non normalized Gaussian density with mean m, and standard deviation s. A sample of the functions obtains by varying the p, parameter. The system operates by testing rules of different types.

The degree of fulfillment for such a rule in this study was chosen to be the minimum of the degrees of fulfillment of the antecedent clauses. The total output of the control system is calculated as weighted sum of the responses to all n rules outputs. The system operates by testing rules of different types.

IF x_i is high AND y_j is low THEN u_i is slow increasing

3. Multilevel fuzzy functions

A modeling approach to parameters selection by multilevel fuzzy functions is developed on the basis of historical data and experience on a considered process. Fizzy set theory is a step toward an approchement between the precision of classical mathematics and the pervasive imprecision of the real world. Fuzziness of a phenomena steams from the lack of clearly defined boundaries.

Let,

$$A, A_i^{'}\ (j=1,2..m) \quad A_i^{'} \subset A \qquad (2)$$

be to output, global observation, set and subsets, which contain various states to be diagnosed. Since output states in complex processes are often inconclusive, fuzzy set and fuzzy subsets A and A'_j are assumed to describes in practice. Assume that the observed field is a measurable output vector space consisting of n vectors:

$$X = (X_1, X_2, X_3, ...X_n) \qquad (3)$$

where X_i is ith vector with which A_j can be ambiguously predicted, i.e $.A_j^{'}$ can be determined according to the values of X_i (i=1,2,..n); j=1,2,..m).

Suppose that m fuzzy subsets are divided into k groups by various characteristics such as the kinds of faults, where p_i have sum equal one.

Any fuzzy subset A'_j of X_i is characterized by a membership functions μ_{A_j} which associates with every member x_i of X_i , i.e., $\mu'_{A_j}(x_i)$ representing the degree of membership of x to fuzzy subset A'_j.

Statement 1. For every fuzzy subset A'_j in the fuzzy set A a limited sequence fq with f-cut is determined , i.e..

$$0 < f_{q_1} < f_{q_2} < ... < f_{q_{p_q-1}} < 1 \qquad (4)$$

The determination of the corresponding membership function must have the following relationship:

IF $f_{qp_q-1} \leq \mu_{qp_q}(x_i) \leq 1$ THEN $x_i \subset A'_{qp_q}$

IF $f_{qp_q-2} \leq \mu_{qp_q-1}(x_i) \leq f_{qp_q-1}$

THEN $x_i \subset A'_{qp_q-1}$

IF $0 \leq \mu_{qp_q}(x_i) \leq f_{q1}$ THEN $x_i \subset A'_{q1}$ (5)

where $x_i \subset A'_q$ means that it satisfied by the condition in which the state A'_q appears. Therefore, the membership functions are divided into several levels, such as μ_q having p_q levels. This is the membership function of the first order and can be denoted by $\mu^1_{A'_j}(x_i)$.

Statement 2. The membership function of the second order can be structured by the composition by several membership function of the first order,

$$\mu^2(x_i) = \sum_{i=d}^{g} w_i \mu^1(x_i) \qquad (6)$$

$$\mu^2(x_i) = \mu^2(x_d, ..., x_g), 1 \leq d < g \leq n$$

where sum of w_i equal one.

Statement 3. The membership function of the

$$\mu^3(x_i) = \sum_{i=l}^{b} w_i \mu^2(x_i) \qquad (7)$$

third order can be structured by the composition of several membership functions of the second order,

where sum of w_i equal one.

In regard to the qth group of parameter (q=1) follows:

IF $f_1 \leq \mu^3(x_i) \leq 1$ THEN selected parameter is true.

4. A fuzzy control system

The control system model is shown in Figure 1. Dynamic characteristics of the plant can be approximated by linear functions and controlled by conventional proportional integral law.

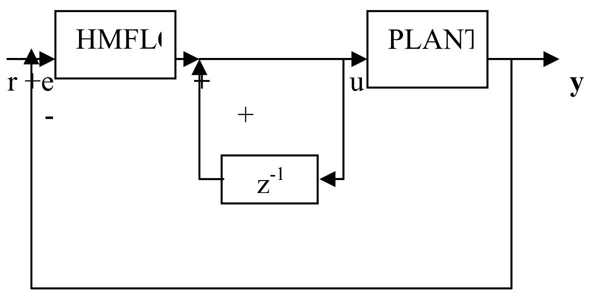

Figure 1. Control system loop

The qualitative model for systematic cause - event analysis was made, and variables discrete state were defined:

Input variables (low, medium, high)

Output variables (low, medium, high)

Control variables (increasing, slow increasing, normal, slow decreasing , decreasing).

5. Clustering fuzzy controller design

The block diagram of the fuzzy control system is shown in Fig. 1. For the simple fuzzy logic controller, which has two inputs and one output, fuzzy model is derived. The error and the change of error are defined as:

$$e(k) = r(k) - y(k) \qquad (8)$$

$$\Delta e(k) = e(k) - e(k+1) \qquad (9)$$

the inputs of the fuzzy controller are the normalized error and the normalized change of error. The membership function μ(**x**) of the fuzzified inputs are of triangular shape. The two fuzzy regions positive (P) and negative (N) for two input variables and the corresponding membership functions are defined for positive fuzzy labels and negative fuzzy labels eqs.(4)-(5).

Consequently, the four simple fuzzy control rules,being

used in a simple fuzzy logic controller as follows in eq.(6).

Using the center of gravity defuzzification method the control output of the type of fuzzy logic controller-FLC can be obtained, when the normalized error and the normalized change of error are inside the universe of discourse.

For the positive fuzzy label,

$$\mu(w_{x_i}) = \begin{cases} 0 & w_{x_i}x_i, -1 \\ 0.5 + w_{x_i}x_i & -1 \le w_{x_i}x_i < 1 \\ 1 & w_{x_i} \ge 1 \end{cases} \qquad (10)$$

for negative fuzzy labels,

$$\mu(w_{x_i}) = \begin{cases} 0 & w_{x_i}x_i, -1 \\ 0.5 - w_{x_i}x_i & -1 \le w_{x_i}x_i < 1 \\ 1 & w_{x_i} \ge 1 \end{cases} \qquad (11)$$

where $x_i = (e, \Delta e)$.

The simple four rules fuzzy controller:

IF (**e** is **N**) AND (**Δe** is **N)** THEN

(change in control is N)

IF (**e** is **N**) AND (**Δe** is **P**) THEN

(change in control is Z)

IF (**e** is **P**) AND (**Δe** is **N**) THEN

(change in control is Z)

IF (**e** is **P**) AND (Δe is P) THEN

(change in control is P). (12)

where the fuzzy labels of the control outputs are

singeltons defined as P=1, Z=0 and N=-1.

The control output for the error and change of error

inside the universe of discourse,

$$\Delta u = \frac{w_{\Delta u} w_{\Delta e}}{4-2a}\left(\Delta e + \frac{\Delta t}{\frac{w_{\Delta e}}{w_e}\Delta t} e\right) = K_c^F\left(\Delta e + \frac{\Delta t}{T_i^F} e\right) \qquad (13)$$

with

$$a = \max(w_e|e|, w_{\Delta e}|\Delta e|) \qquad (14)$$

where , $x_i = (e, \Delta e)$, $w_{\Delta u}$ is the scaling factor of the fuzzy control output. It can conclude tht this kind of basic fuzzy logic controller is a linear proportional-integral controller in structure, with a nonlinear proportional gain $K_c^{(F)}$, and integral $T_i^{(F)}$ when inside the universe of discourse.

In the design method based on gain and phase margins, it is also important to select a suitable equivalent gain/phase margin contour, and the improper allocation of the equivalent contour degrade the system performance. The overcome this difficulty, clustering fuzzy rules are included in the gain /phase margin tuning method, such that the equivalent gain phase margin contour can be simply fixed.

In regard to q –th group of parameters (q=1)follows:

IF $f_1 \le \mu^2(x_i) \le 1$

THEN **gain** is setting, (16)

IF $0 \le \mu^2(x_i) \le f_1$

THEN **phase** is setting. (15)

These two production rules, Eq.(9)-(10), improve

design parameter selection.

In regard to the qth group of parameter (q=1) follows:

IF $f_1 \le \mu^3(x_i) \le 1$

THEN gain/phase margin is accurate. (16)

This production rule, Eq.(11), shows proper allocation of the equivalent gain /phase margin contour.

6. Description of the case study

The case study consists of two coupled stirred tanks as shown in Figure 2. The basic task of the experiment is to control the mixture level in the second stirred tank of the coupled system. The basic control strategy is to control the mixture level in the second tank by varying the input flow to the first tank. The measurement for liquid level is read in, and control signal is written out

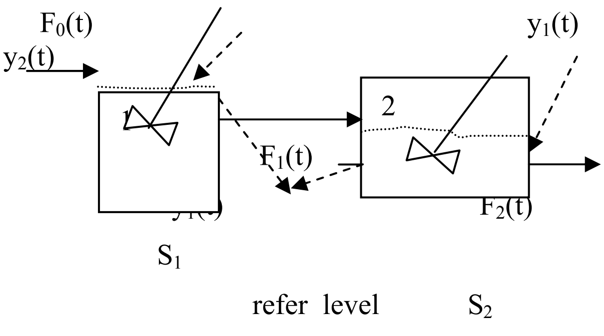

Figure 2. Scheme of the plant

The dynamic model of the plant can be defined as:

$$\rho S_1 \frac{dy_1(t)}{dt} = F_0(t) - F_1(t) \qquad (17)$$

$$\rho S_2 \frac{dy_2(t)}{dt} = F_1(t) - F_2(t) \qquad (18)$$

where $y_1(t)$ and $y_2(t)$ are tanks levels in the first and second tank, respectively measured from refer levels: $y_1(0) = y_2(0) = 0$, $F_0(t)$, $F_1(t)$ and $F_2(t)$ are flow rates, and S_1, and S_2 are cross section. $F_1(t)$ can express as $f_1(t)=ky_1(t)$.

The plant could be modeled by transfer functions in matrix form:

$$G^{-1}(s)\,\bar{y}(s) = \bar{x}(s) \qquad (19)$$

where
$$G^{-1}(s) = \begin{vmatrix} \rho_1 S_1 s + k & 0 \\ -k & \rho S_2 s + k \end{vmatrix}$$

$$\bar{y}(s) = \begin{vmatrix} y_1 \_s \\ y_2 - s \end{vmatrix} \text{ and } \bar{x}(s) = \begin{vmatrix} F_1(t) & 0 \\ -F_2(t) & 0 \end{vmatrix}$$

It is easy to find out that this design parameter is related to certain points on the normalized phase plane and this is can be interpreted as an equivalent gain/phase margin contour (Figure 3). α_0 is design parameter which can be selected from 0-1. Since,

$\alpha_0=\max(w_e|e_0|, w_{\Delta e}|\Delta e|)$. (20)

According to the tuning formula, the weighting factors $w_e, w_{\Delta e}, w_{\Delta u}$ will be fixed and the gain of the fuzzy logic controller when the system is at its contour can be determined.

Clearly, the fuzzy controller has the property that its is fixed and is variable in terms of different e and Δe. The property of ao can be used to allocate the equivalent gain/phase margin contour so as to modify the

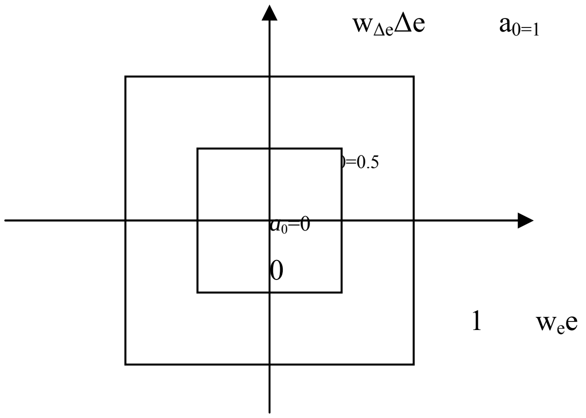

Figure 3 Equivalent gain/phase margin for α_0 from 0 to 1

closed loop system's performance. The multilevel fuzzy controller goes further in sensitivity varying gain or phase, alternatively according to equations (16)-(17) in terms of different e and Δe. The higher order multilevel fuzzy logic controller-HMFLC shows better robustness and sharpness to allocate the equivalent gain/phase margin contour, according to equation (18) so as to modify the closed loop system performance.

The main contribution of this paper is the clustering of a higher order multilevel fuzzy logic controller. The membership function third order defines common action of design parameters.

The resulting HMFLC appears to be a varying parameter controller which is a realization of the fuzzy control rules equations (13) and fuzzy parameters selection rules equations (16) to (18) as a clustering system.The switching line of the fuzzy control action on is:

$$w_e e = w_{\Delta u} \quad (21)$$

On the switching line the control action is zero. When the system state is across the switching line, the control action will change its sign.

7. Results and discussion

The multilevel tuning actually has made the fuzzy logic controller able to work on cluster manner and to improve adaptation of the control environment.

The dynamic response and control for the step disturbance to the inlet flow rate in the first tank $f_0(t)$ was studied. The obtained results are shown in Figures 5 and 6.

An inverse model in Figure 4 was used.

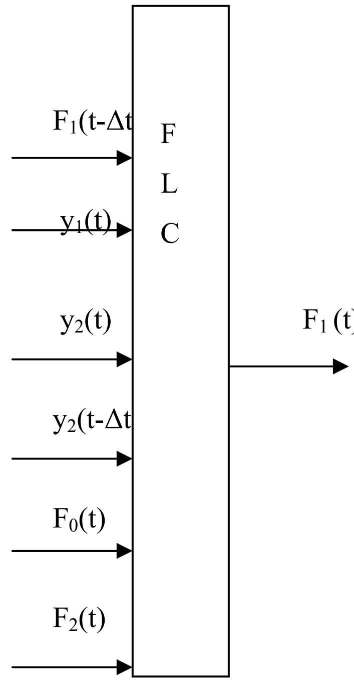

Figure 4. An inverse model control

The fuzzy rules control system:

IF $F_o(t)$ is medium AND $F_1(t)$ is low THEN

y_2 (t) is decreasing.

IF $F_1(t)$ is high AND $F_2(t)$ is low THEN

y_2 (t) is increasing

IF $F_1(t)$ is medium AND y_1 (t) is high THEN

$y_2(t)$ is slow increasing.

IF F_1 (t) is low AND $y_1(t)$ is low THEN

$y_2(t)$ is decreasing.

IF $f_1 \le \mu^2{}_{y_1(t)}(F_0(t)) \le 1$ AND $0 \le \mu^2{}_{y_2(t)}(F_0(t)) \le f_1$ THEN $y_1(t)$ is setting

IF $0 \le \mu^2{}_{y_1(t)}(F_0(t)) \le f_1$ AND $f_1 \le \mu^2{}_{y_2(t)}(F_0(t)) \le 1$ THEN $y_2(t)$ is setting

The investigation is carried out during a time period from 0 to 1200s. The input flow is supplied varying the flow to the first tank. First, the system is settled at 3 cm.

There are set point changes from 3cm to 6cm and 10 to 11cm at time instants of 30, and 120 respectively.

Moreover, there are one load disturbance at time instants at 90 and 270 which is $10 cm^3/s$. The specified gain and phase margins is 2.5 and 45˚, respectively. The sampling interval is $\Delta t = 1 s$.

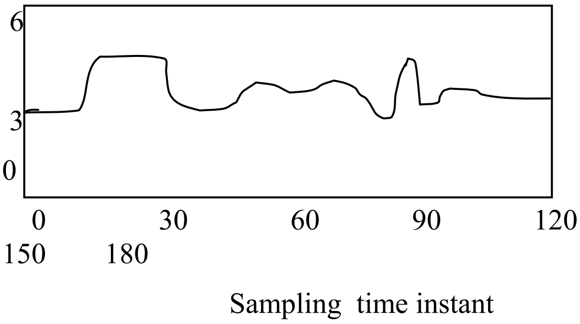

Figure 5. Tank 1 level $y_1(t)$response for $F_0(t)$ step disturbance

The fuzzy controller makes the closed loop system a bit sluggish. Multilevel fuzzy tuning algorithm, the closed loop system has only a slight overshoot, and converges quickly to the set point. The values of the second order membership functions show which parameters can be fixed or varied. For example, if the value of membership function second order for gain margin is higher from the value of membership function second order for phase margin then gain is varied and phase margin is fixed. The improvement higher order multilevel order tuning algorithm is in proper allocation of the equivalent gain/phase margin contour. The membership function third order is determined gain/phase margin resolution.

These rules and formula are helpful in eliminating the most time consuming trial-and-error procedures in the synthesis and design of fuzzy control systems. A control effort will be partly or fully saturated outside the universe of discourse. It has shown that the weighting factors are functions of both parameters of the

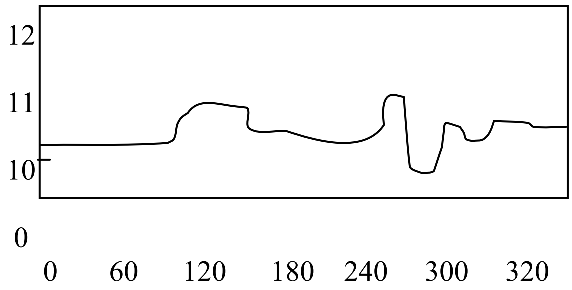

Figure 6. Tank 2 level $y_2(t)$ to $F_0(t)$ step disturbance at the time instant 90 and 270

plant under control and the performance index of the closed loop system.

In the clustering method based on gain and phase margins, it is also important to select suitable equivalent gain/phase margin contour so as to obtain appreciable performance.

8.Conclusion

The obtained results are shown the system reaches the set point faster with less overshoot, hence the settling time is the shortest, especially for unstable regions. The underlying idea of the multilevel fuzzy logic controller is to associate the integral action and the proportional action of the fuzzy logic controller with the normalized dead time and state error. The higher order multilevel fuzzy logic controller shows higher robustness and sharpness, and improves proper allocation of the suitable equivalent gain/phase margin contour. The improper allocation of the equivalent contours degrade the system performance. This paper is the first report in the literature showing the clustering fuzzy controller tuning. The higher order multilevel controller was improved robustness and sharpness to determine gain/phase margin a well as parameters selectivity and can be applied in flexible control systems. The obtained results in this paper could be used in the other domain.
Acknowledgment. The author wishes to express her gratitude to the Fund of Serbia for financial support.

Notation

A - fuzzy subset
A - fuzzy set
e - error
Δe - change of error
f - experienced factor (0.8)
k - number of level
N - negative fuzzy label
q - number of sequences
P -positive fuzzy label
s - set point
t, T_i -time, integral time, respectively
K -proportional gain
u -fuzzy control output
x -input variable
y -output variable
w -weighting factor

Subscript
(F) - fuzzy
Index
c -control
e - error
Δe - change of error
i - integral
q - number of sequences (fragments)
p - crossover frequency

Greek Symbols
αo - design parameters
μ^1(..)-fuzzy membership function of the first order
μ^2(..)-fuzzy membership function of the second order
μ^3(..)-fuzzy membership function of the third order

References

Savkovic-Stevanovic J. (2004) The higher order multilevel fuzzy logic controller, Chem.Biochem. Eng.**Q**.**18**, 345-352.

Xu.,J.H.,C. Liu and C-C.Hang (1996) Modified tuning of a of a fuzzy logic controller**,** Engng.Applic.Artif. Intell**.** **9**, 65-74.

Savkovic-Stevanovic J., M.Ivanovic (2000) Tuning of a fuzzy logic controller, ISCAPE2000-The firs International Symposium on Computer Aided Process Engineering, Cartagena de Indias, Colombia, January 24-28, P.F4.

Hang.C.C., W.K.Ho and L.S.Cao, ISA Trans. **33** (1994) 147.

Ho.W.K.,C.C.Hang and L.S.Cao (1995) Tuning of PID controllers based on gain and phase margins specifications, Automatica**, 31** (1995) 497-502.

Chunsheng,F.W. Shuqing and W.Jicheng (1988) A modeling approach to trouble diagnosis by multilevel fuzzy function and its applications, The 4th ICAFT- Int.Congrss on Comp. Appl. Ind. Ferm. Techn.,SCI, Camridge, U.K., Sep.25-29.pp.388.

Savkovic-Stevanovic, J.,W. D.Seider, L.Ungar, A fuzzy logic controller for pH control of a chemical stirred tank, Chem. Ind. (Belgrade), **54** (2000) 384-388.

Savkovic-Stevanovic J.(2000) Multivalued, multilevel fuzzy functions for faults recognition and classification, IES'2000-The 11the Seminar and Symposium on Information and Expert Systems in the Process Industries**,** Belgrade, October 27-28,pp. 83-98.

IZVOD

REGULACIONI SISTEM U USLOVIMA NEIZVESNOSTI SA VIŠE PROMENLJIVIH I NA VIŠE NIVOA

Jelenka Savković-Stevanović

Tehnološko-metalurški fakultet Univerziteta u Beogradu, Karnegijeva 4, 11000 Beograd, Srbija

U ovom radu dat je fazi regulacioni sistem višeg reda sa više nivoa. Razvijena je metoda prebacivanja sa funkcijama pripadnosti drugog i trećeg reda. Fazi regulacija višeg reda i na više nivoa izvedena je tako da varira u odnosu na normalizovano vreme kašnjenja postrojenja i sistemske greške. Kao slučaj za ispitivanje korišćeno je postrojenje od dva spojena rezervoara. Simulacija i eksperimenti su dati da pokažu validnost predložene metode prebacivanja. Dobiveni rezultati pokazuju prednosti fazi regulatora višeg reda sa više nivoa u preciznosti i selektivnosti parametara. Fazi regulator višeg reda sa više nivoa postavlja se brže od fazi regulatorom prvog reda u konjukciji sa manjim problemima u postavci početnih tačaka regulacije. Dobiveni rezultati ilustruju fleksibilnost u korišćenju fazi regulatora trećeg reda sa više nivoa u formuli prebacivanja.
Ključne reči: Regulacija u uslovima neizvesnosti na više nivoa, fazi regulator višeg nivoa, normalizovano vreme kašnjenja, greška stanja.